

Greybody factor for a scalar field coupling to Einstein's tensor

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Abstract

We study the greybody factor and Hawking radiation for a scalar field coupling to Einstein's tensor in the background of Reissner-Nordström black hole in the low-energy approximation. We find that the presence of the coupling terms modifies the standard results in the greybody factor and Hawking radiation. Our results show that both the absorption probability and Hawking radiation increase with the coupling constant. Moreover, we also find that for the stronger coupling, the charge of black hole enhances the absorption probability and Hawking radiation of the scalar field, which is different from those of ones without coupling to Einstein's tensor in the black hole spacetime.

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I. INTRODUCTION

Scalar field, associated with spin-0 particles in quantum field theory, has been an object of great interest for physicists in the latest years. One of the main reasons is that the models with scalar fields are relatively simple, which allows us to probe the detailed features of the more complicated physical system. In cosmology, scalar fields can be considered as candidate to explain the inflation of the early Universe [1] and the accelerated expansion of the current Universe [2–4]. In the Standard Model of particle physics, the scalar field presents as the Higgs boson [5], which would help to explain the origin of mass in the Universe. Moreover, it has been found that scalar field plays the important roles in other fundamental physical theories, such as, Jordan-Brans-Dicke theory [6], Kaluza-Klein compactification theory [7] and superstring theory [8], and so on.

In general, the action contained scalar fields in Einstein's theory of gravity is

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + \frac{1}{2} \partial_\mu \psi \partial^\mu \psi + V(\psi) + \xi R \psi^2 \right] + S_m, \quad (1)$$

where ψ , R and $V(\psi)$ are corresponding to scalar field, Ricci scalar and scalar potential, respectively. The term $\xi R \psi^2$ represents the coupling between Ricci scalar R and the scalar field ψ . The dynamical behavior of the scalar field in the theory (1) have been investigated very extensively in the modern physics including cosmology and black hole physics. The more general form of the action contained scalar field in other theories of gravity is

$$S = \int d^4x \sqrt{-g} \left[f(\psi, R, R_{\mu\nu} R^{\mu\nu}, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}) + K(\psi, \partial_\mu \psi \partial^\mu \psi, \nabla^2 \psi, R^{\mu\nu} \partial_\mu \psi \partial_\nu \psi, \dots) + V(\psi) \right] + S_m, \quad (2)$$

Here f and K are arbitrary functions of the corresponding variables. Obviously, the more coupling between scalar field and curvature are considered in these extensive theories. The non-minimal coupling between the derivative of scalar field and the spacetime curvature may appear firstly in some Kaluza-Klein theories [9–11]. Amendola [12] considered the most general theory of gravity with the Lagrangian linear in the Ricci scalar, quadratic in ψ , in which the coupling terms have the forms as follows

$$R \partial_\mu \psi \partial^\mu \psi, \quad R_{\mu\nu} \partial^\mu \psi \partial^\nu \psi, \quad R \psi \nabla^2 \psi, \quad R_{\mu\nu} \psi \partial^\mu \psi \partial^\nu \psi, \quad \partial_\mu R \partial^\mu \psi, \quad \nabla^2 R \psi. \quad (3)$$

And then he studied the dynamical evolution of the scalar field in the cosmology by considering only the derivative coupling term $R_{\mu\nu} \partial^\mu \psi \partial^\nu \psi$ and obtained some analytical inflationary solutions [12]. Capozziello *et al.* [13, 14] investigated a more general model of containing coupling terms $R \partial_\mu \psi \partial^\mu \psi$ and $R_{\mu\nu} \partial^\mu \psi \partial^\nu \psi$, and found that the de Sitter spacetime is an attractor solution in the model. Recently, Daniel and Caldwell [15]

obtained the constraints on the theory with the derivative coupling term of $R_{\mu\nu}\partial^\mu\psi\partial^\nu\psi$ by Solar system tests. In general, a theory with derivative couplings could lead to that both the Einstein equations and the equation of motion for the scalar are the fourth-order differential equations. However, Sushkov [16] studied recently the model in which the kinetic term of the scalar field only coupled with the Einstein tensor and found that the equation of motion for the scalar field can be reduced to second-order differential equation. This means that the theory is a “good” dynamical theory from the point of view of physics. Gao [17] investigated the cosmic evolution of a scalar field with the kinetic term coupling to more than one Einstein tensors and found the scalar field presents some very interesting characters. He found that the scalar field behaves exactly as the pressureless matter if the kinetic term is coupled to one Einstein tensor and acts nearly as a dynamic cosmological constant if it couples with more than one Einstein tensors. The similar investigations have been considered in Refs.[18, 19]. These results will excite more efforts to be focused on the study of the scalar field coupled with tensors in the more general cases.

Since black hole is another fascinating object in modern physics, it is of interest to extend the study of the properties of the scalar field when it is kinetically coupled to the Einstein tensors in the background of a black hole. In this Letter, we will investigate the greybody factor and Hawking radiation of the scalar field coupling only to the Einstein tensor $G^{\mu\nu}$ in the Reissner-Nordström black hole spacetime by using the matching technique, which has been widely used in evaluating the absorption probabilities and Hawking radiations of various black holes [20–29]. We find that the presence of the coupling terms enhances both the absorption probability and Hawking radiation of the scalar field in the black hole spacetime. Moreover, we also find that for the stronger coupling, the absorption probability and Hawking radiation of the scalar field increase with the charge of the black hole, which is different from those of scalar one without coupling to Einstein’s tensor.

The Letter is organized as follows: in the following section we will introduce the action of a scalar field coupling to Einstein’s tensor and derive its master equation in the Reissner-Nordström black hole spacetime. In Sec.III, we obtain the expression of the absorption probability in the low-energy limit by using the matching technique. In section IV, we will calculate the absorption probability and the luminosity of Hawking radiation for the coupled scalar field. Finally, in the last section we will include our conclusions.

II. MASTER EQUATION FOR A SCALAR FIELD COUPLING WITH EINSTEIN'S TENSOR IN THE CHARGED BLACK HOLE SPACETIME

Let us consider the action of the scalar field coupling to the Einstein's tensor $G^{\mu\nu}$ in the curved spacetime [16],

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + \frac{1}{2} \partial_\mu \psi \partial^\mu \psi + \frac{\eta}{2} G^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \right]. \quad (4)$$

The coupling between Einstein's tensor $G^{\mu\nu}$ and the scalar field ψ is represented by the term $\frac{\eta}{2} G^{\mu\nu} \partial_\mu \psi \partial_\nu \psi$, where η is coupling constant with dimensions of length-squared. In general, the presence of such a coupling term brings some effects to the original metric of the background. However, it is very difficult for us to obtain an analytic solution for the action (4). Actually, comparing with the mass of the black hole, one can find that the energy of a scalar field is very tiny so that its back-reaction effects on the background can be neglected exactly. Here, we treat the weak external scalar field as a probe field, and then study the effects of the coupling constant η on the greybody factor and Hawking radiation of the scalar field in the background of a black hole spacetime.

Varying the action with respect to ψ , one can obtain the modified Klein-Gordon equation

$$\frac{1}{\sqrt{-g}} \partial_\mu \left[\sqrt{-g} \left(g^{\mu\nu} + \eta G^{\mu\nu} \right) \partial_\nu \psi \right] = 0, \quad (5)$$

which is a second order differential equation. Obviously, all the components of the tensor $G^{\mu\nu}$ vanish in the Schwarzschild black hole spacetime because it is the vacuum solution of the Einstein's field equation. Thus, we cannot probe the effect of the coupling term on the greybody factor and Hawking radiation in the background of a Schwarzschild black hole. The simplest black hole with the non-zero components of the tensor $G^{\mu\nu}$ is Reissner-Nordström one, which can be described by the metric as follows

$$ds^2 = -f dt^2 + \frac{1}{f} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (6)$$

with

$$f = 1 - \frac{2M}{r} + \frac{q^2}{r^2}, \quad (7)$$

where M is the mass and q is the charge of the black hole. The Einstein's tensor $G^{\mu\nu}$ for the metric (6) has a form

$$G^{\mu\nu} = \frac{q^2}{r^4} \begin{pmatrix} -\frac{1}{f} & & & \\ & f & & \\ & & -\frac{1}{r^2} & \\ & & & -\frac{1}{r^2 \sin^2 \theta} \end{pmatrix}. \quad (8)$$

Adopting to the spherical harmonics $\psi(t, r, \theta, \phi) = e^{-i\omega t} R(r) Y_{lm}(\theta, \phi)$, we find that the equation (5) can be separable and the radial function $R(r)$ obeys to

$$\frac{1}{r^2} \frac{d}{dr} \left[r^2 \left(1 + \frac{\eta q^2}{r^4} \right) f \frac{dR(r)}{dr} \right] + \left[\left(1 + \frac{\eta q^2}{r^4} \right) \frac{\omega^2}{f} - \left(1 - \frac{\eta q^2}{r^4} \right) \frac{E_{lm}}{r^2} \right] R(r) = 0, \quad (9)$$

where $E_{lm} = l(l+1)$ is the eigenvalues coming from the angular equation. Clearly, the radial equation (9) contains the coupling constant η , which means that the presence of the coupling term will change the evolution of the scalar field in the Reissner-Nordström black hole spacetime. Moreover, as the charge q vanishes, it is easy to obtain that the effects of the coupling constant η disappears and the radial equation (9) reduces to that in the Schwarzschild black hole spacetime.

The solution of the radial function $R(r)$ will help us to obtain the absorption probability $|A_l|^2$ and the luminosity of Hawking radiation for a scalar field coupling to Einstein's tensor in the Reissner-Nordström black hole spacetime.

III. GREYBODY FACTOR IN THE LOW-ENERGY REGIME

In this section, we will present an analytic expressions for the greybody factors for the emission of a scalar field coupling to Einstein's tensor in the Reissner-Nordström black hole spacetime. Since the radial equation (9) is generally nonlinear, it is very difficult to obtain its analytic solution. However, following Refs.[20–29], we can provide an approximated solution of the radial equation (9) by employing the matching technique. Firstly, we can derive the analytic solutions in the near horizon ($r \simeq r_+$) and far-field ($r \gg r_+$) regimes in the low-energy limit (i.e., $\omega \ll T_H$ and $\omega r_+ \ll 1$), respectively. Then, we can match smoothly these two radial functions $R(r)$ in an intermediate region since the wave function in physics is continuous everywhere. Through this matching technique we can construct a smooth analytical solution of the radial equation valid throughout the entire spacetime and extract further the ratio between two coefficients $A_{out}^{(\infty)}$ and $A_{in}^{(\infty)}$ in Eq. (31), which helps us define the greybody factor. The main reason that we here adopt to the low-energy approximation is that in this limit we can neglect the back-reaction of a scalar field to the background metric during the emission process. Moreover, it is well known that the greybody factor modifies the spectrum of emitted particles from that of a perfect thermal blackbody. However, in the high-energy regime, the greybody factor is independent of the energy ω of the particle and the spectrum is exactly like that of a perfect blackbody for every particle species [27], while in the low-energy regime, it encodes information about the near horizon structure of a black hole and about the particles emitted by the black hole.

Now, we focus on the near-horizon regime and perform the following transformation of the radial variable as in Refs. [27–29]

$$r \rightarrow f \Rightarrow \frac{df}{dr} = (1-f) \frac{\mathcal{A}}{r}, \quad (10)$$

with

$$\mathcal{A} = 1 - \frac{q^2}{2Mr - q^2}. \quad (11)$$

The equation (9) near the horizon ($r \sim r_+$) can be rewritten as

$$f(1-f) \frac{d^2 R(f)}{df^2} + (1-D_* f) \frac{dR(f)}{df} + \left[\frac{K_*^2}{\mathcal{A}(r_+)^2(1-f)f} - \frac{E_{lm}}{\mathcal{A}(r_+)^2(1-f)} \left(\frac{r_+^4 - \eta q^2}{r_+^4 + \eta q^2} \right) \right] R(f) = 0, \quad (12)$$

where

$$K_* = \omega r_+, \quad D_* = 1 - \frac{2q^2 r_+^2 (r_+^4 - 2\eta r_+^2 + 3\eta q^2)}{(r_+^2 - q^2)^2 (r_+^4 + \eta q^2)}. \quad (13)$$

Making the field redefinition $R(f) = f^\alpha (1-f)^\beta F(f)$, one can find that the equation (12) can be rewritten as a form of the hypergeometric equation

$$f(1-f) \frac{d^2 F(f)}{df^2} + [c - (1+a+b)f] \frac{dF(f)}{df} - abF(f) = 0, \quad (14)$$

with

$$a = \alpha + \beta + D_* - 1, \quad b = \alpha + \beta, \quad c = 1 + 2\alpha. \quad (15)$$

Considering the constraint coming from coefficient of $F(f)$, one can easy to obtain that the power coefficients α and β satisfy

$$\alpha^2 + \frac{K_*^2}{\mathcal{A}(r_+)^2} = 0, \quad (16)$$

and

$$\beta^2 + \beta(D_* - 2) + \frac{1}{\mathcal{A}(r_+)^2} \left[K_*^2 - E_{lm} \left(\frac{r_+^4 - \eta q^2}{r_+^4 + \eta q^2} \right) \right] = 0, \quad (17)$$

respectively. These two equations admit that the parameters α and β have the forms

$$\alpha_\pm = \pm \frac{iK_*}{\mathcal{A}(r_+)}, \quad (18)$$

$$\beta_\pm = \frac{1}{2} \left\{ (2 - D_*) \pm \sqrt{(D_* - 2)^2 - \frac{4}{\mathcal{A}(r_+)^2} \left[K_*^2 - E_{lm} \left(\frac{r_+^4 - \eta q^2}{r_+^4 + \eta q^2} \right) \right]} \right\}, \quad (19)$$

Following the operation in ref.[27–29] and using the boundary condition that no outgoing mode exists near the horizon, we can obtain that the parameters $\alpha = \alpha_-$ and $\beta = \beta_-$. Thus the asymptotic solution near horizon has the form

$$R_{NH}(f) = A_- f^\alpha (1-f)^\beta F(a, b, c; f), \quad (20)$$

where A_- is an arbitrary constant.

Let us now to stretch smoothly the near horizon solution to the intermediate zone. As done in ref.[27–29], we can make use of the property of the hypergeometric function [30] and change its argument in the near horizon solution from f to $1-f$

$$\begin{aligned} R_{NH}(f) = & A_- f^\alpha (1-f)^\beta \left[\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F(a, b, a+b-c+1; 1-f) \right. \\ & \left. + (1-f)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F(c-a, c-b, c-a-b+1; 1-f) \right]. \end{aligned} \quad (21)$$

As $r \gg r_+$, the function $(1-f)$ can be approximated as

$$1-f = \frac{2Mr - q^2}{r^2} \simeq \frac{2M}{r}, \quad (22)$$

and then the near horizon solution (21) can be simplified further to

$$R_{NH}(r) \simeq C_1 r^{-\beta} + C_2 r^{\beta+D_*-2}, \quad (23)$$

with

$$C_1 = A_- (2M)^\beta \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \quad (24)$$

$$C_2 = A_- (2M)^{-(\beta+D_*-2)} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}. \quad (25)$$

In order to obtain a solution in the far field region, we expand the wave equation (9) as a power series in $1/r$ and keep only the leading terms

$$\frac{d^2 R_{FF}(r)}{dr^2} + \frac{2}{r} \frac{dR_{FF}(r)}{dr} + \left(\omega^2 - \frac{E_{lm}}{r^2} \right) R_{FF}(r) = 0. \quad (26)$$

This is usual Bessel equation. Thus the solution of radial master equation (9) in the far-field limit can be expressed as

$$R_{FF}(r) = \frac{1}{\sqrt{r}} \left[B_1 J_\nu(\omega r) + B_2 Y_\nu(\omega r) \right], \quad (27)$$

where $J_\nu(\omega r)$ and $Y_\nu(\omega r)$ are the first and second kind Bessel functions, $\nu = l + \frac{1}{2}$. B_1 and B_2 are integration constants. In order to stretch the far-field solution (27) towards small radial coordinate, we take the limit $r \rightarrow 0$ and obtain

$$R_{FF}(r) \simeq \frac{B_1(\frac{\omega r}{2})^\nu}{\sqrt{r} \Gamma(\nu + 1)} - \frac{B_2 \Gamma(\nu)}{\pi \sqrt{r} (\frac{\omega r}{2})^\nu}. \quad (28)$$

In the low-energy limit, the two power coefficients in Eq.(23) can be approximated as

$$\begin{aligned} -\beta &\simeq l + O(\omega^2), \\ (\beta + D_* - 2) &\simeq -(l + 1) + O(\omega^2), \end{aligned} \quad (29)$$

respectively. By matching the corresponding coefficients between Eqs.(23) and (28), we can obtain two relations between C_1 , C_2 and B_1 , B_2 . Removing A_- , we can obtain the ratio between the coefficients B_1 , B_2

$$\begin{aligned} B \equiv \frac{B_1}{B_2} &= -\frac{1}{\pi} \left[\frac{1}{\omega M} \right]^{2l+1} \left(\frac{2l+1}{2} \right) \Gamma^2(l + \frac{1}{2}) \\ &\times \frac{\Gamma(c-a-b)\Gamma(a)\Gamma(b)}{\Gamma(a+b-c)\Gamma(c-a)\Gamma(c-b)}. \end{aligned} \quad (30)$$

In the asymptotic region $r \rightarrow \infty$, the solution in the far-field can be expressed as

$$\begin{aligned} R_{FF}(r) &\simeq \frac{B_1 + iB_2}{\sqrt{2\pi} \omega r} e^{-i\omega r} + \frac{B_1 - iB_2}{\sqrt{2\pi} \omega r} e^{i\omega r} \\ &= A_{in}^{(\infty)} \frac{e^{-i\omega r}}{r} + A_{out}^{(\infty)} \frac{e^{i\omega r}}{r}. \end{aligned} \quad (31)$$

The absorption probability can be calculated by

$$|\mathcal{A}_l|^2 = 1 - \left| \frac{A_{out}^{(\infty)}}{A_{in}^{(\infty)}} \right|^2 = 1 - \left| \frac{B - i}{B + i} \right|^2 = \frac{2i(B^* - B)}{BB^* + i(B^* - B) + 1}. \quad (32)$$

Inserting the expression of B (30) into Eq.(32), we can probe the properties of absorption probability for the scalar field coupled with Einstein's tensor in the charged black hole spacetime in the low-energy limit.

IV. THE ABSORPTION PROBABILITY AND HAWKING RADIATION OF SCALAR FIELD COUPLING TO EINSTEIN'S TENSOR

We are now in a position to calculate the absorption probability and discuss Hawking radiation of a scalar field coupling to Einstein's tensor in the background of a Reissner-Nordström black hole.

In Fig.1, we fixed the coupling constant η and plotted the change of the absorption probability of a scalar particle with the charge q for the first partial waves ($l = 0$) in the Reissner-Nordström black hole. One can easily see that with for the smaller η the absorption probability $A_{l=0}$ decreases with the charge q of black

hole, which is similar to that for the usual scalar field without coupling to Einstein's tensor. However, for the larger η , the absorption probability $A_{l=0}$ increases as the charge q increase, which means that the stronger coupling between the scalar field and Einstein's tensor changes the properties of the absorption probability of scalar field in the black hole spacetime. This phenomenon has not been observed elsewhere. From Fig.2, we also find that the absorption probability increases with the increase of the coupling constant η for fixed values of charge $q = 0.2$. These results about the absorption probability hold true for other values of l . It is also shown in Figs. 3 and 4, in which we plotted the dependence of the absorption probability on the angular index. Moreover, we see the suppression of $|A_l|^2$ as the values of the angular index increase. This can be explained by a fact that the absorption probability $|A_l|^2 \approx \omega^{2l+2}$ in the low-energy approximation $\omega r_+ \ll 1$ [27], which means that the first partial wave dominates over all others in the absorption probability. It is similar to that of the scalar field without coupling to Einstein's tensor as shown in Refs.[20–29].

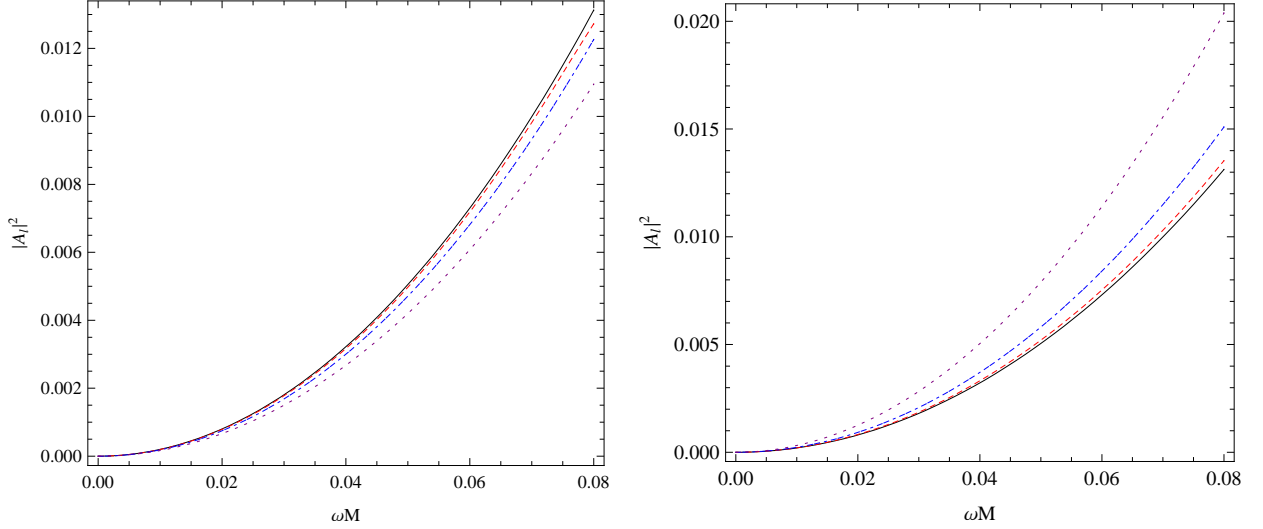


FIG. 1: Variety of the absorption probability $|A_l|^2$ of a scalar field with the charge q in the Reissner-Nordström black hole for fixed $l = 0$. The coupling constant η is set by $\eta = 0.1$ in the left and by $\eta = 1.2$ in the right. The solid, dashed, dash-dotted and dotted lines are corresponding to the cases with $q = 0, 0.1, 0.2, 0.3$, respectively. We set $2M = 1$.

Now let us turn to study the luminosity of the Hawking radiation of a scalar field coupling to Einstein's tensor in the background of a Reissner-Nordström black hole. From above discussion, we know that in the low-energy approximation $\omega r_+ \ll 1$ the mode $l = 0$ plays a dominant role in the greybody factor, which means that in the scalar emission the major contribution comes from the mode $l = 0$ since the power emission spectra of a scalar particle is in direct proportion to $|A_l|^2$. Thus, we here consider only the zero mode ($l = 0$) and study the effect of the coupling constant η on the luminosity of the Hawking radiation in the background spacetime. Performing an analysis similar to that in [27–29], we can obtain that the greybody factor (32) in

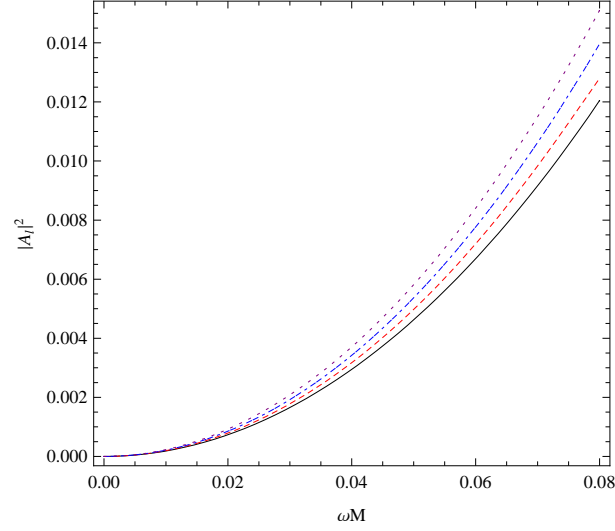


FIG. 2: The dependence of the absorption probability $|A_l|^2$ of a scalar field on the coupling constant η in the Reissner-Nordström black hole for fixed $l = 0$ and $q = 0.2$. The solid, dashed, dash-dotted and dotted lines are corresponding to the cases with $\eta = 0, 0.4, 0.8, 1.2$, respectively. We set $2M = 1$.

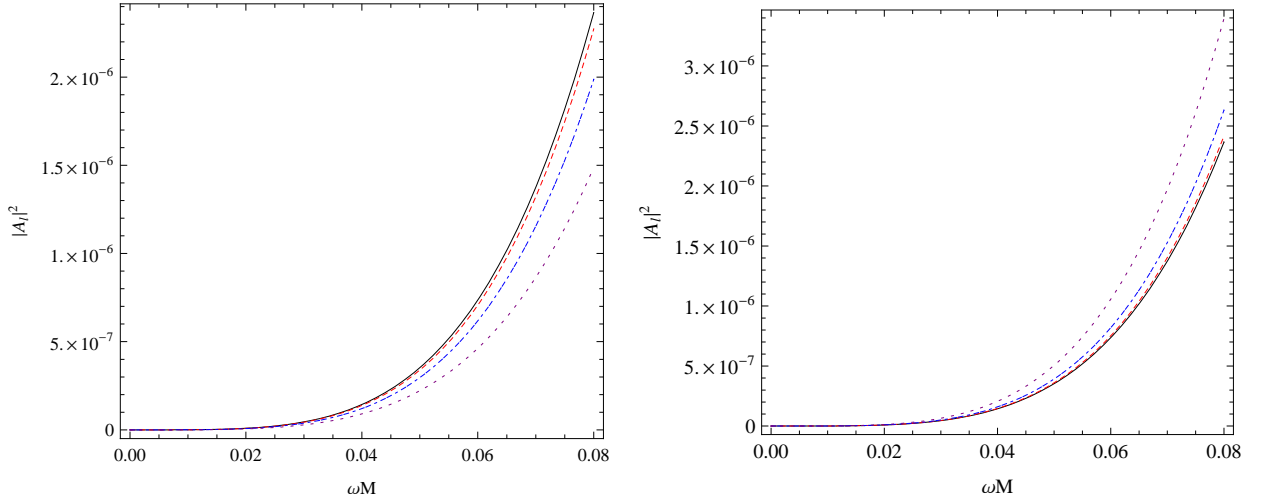


FIG. 3: Variety of the absorption probability $|A_l|^2$ of a scalar field with the charge q in the Reissner-Nordström black hole for fixed $l = 1$. The coupling constant η is set by $\eta = 0.1$ in the left and by $\eta = 1.2$ in the right. The solid, dashed, dash-dotted and dotted lines are corresponding to the cases with $q = 0, 0.1, 0.2, 0.3$, respectively. We set $2M = 1$.

the low-energy limit has a form

$$|A_{l=0}|^2 \simeq \frac{4\omega^2 r_+^2}{\mathcal{A}(r_+)(2 - D_*)}. \quad (33)$$

Combining it with Hawking temperature T_H of Reissner-Nordström black hole, the luminosity of the Hawking radiation for the scalar field with coupling to Einstein's tensor is given by

$$L = \int_0^\infty \frac{d\omega}{2\pi} |A_{l=0}|^2 \frac{\omega}{e^{\omega/T_H} - 1}. \quad (34)$$

The integral expressions above are just for the sake of completeness by writing the integral range from 0 to

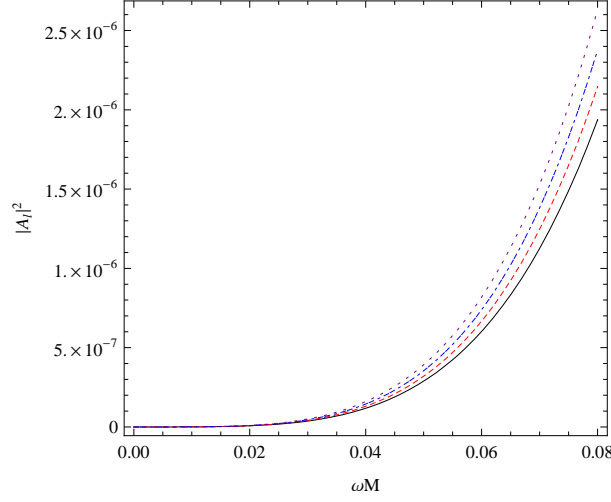


FIG. 4: The dependence of the absorption probability $|A_l|^2$ of a scalar field on the coupling constant η in the Reissner-Nordström black hole for fixed $l = 1$ and $q = 0.2$. The solid, dashed, dash-dotted and dotted lines are corresponding to the cases with $\eta = 0, 0.4, 0.8, 1.2$, respectively. We set $2M = 1$

infinity. However, as our analysis has focused only in the low-energy regime of the spectrum, an upper cutoff will be imposed on the energy parameter so that the low-energy conditions $\omega \ll T_H$ and $\omega r_+ \ll 1$ are satisfied. Like in the radiation of black body, the contribution from the particles with higher frequencies is very tiny in the power spectra of Hawking radiation. Thus, the luminosity of the Hawking radiation for the mode $l = 0$ can be approximated as

$$L \approx \frac{2\pi^3}{15} G T_H^4, \quad (35)$$

with

$$G = \frac{r_+^4 (r_+^2 - q^2)(r_+^4 + \eta q^2)}{(r_+^2 - q^2)^2 (r_+^4 + \eta q^2) + 2q^2 r_+^2 (r_+^4 + 3\eta q^2 - 2\eta r_+^2)}. \quad (36)$$

In Fig.5 and 6, we show the dependence of the luminosity of Hawking radiation (35) on the charge q and the coupling constant η , respectively. From Fig.5, one can easily obtain that with increase of q the luminosity of Hawking radiation L decreases for the smaller η and increases for the larger η , which is similar to the behavior of the absorption probability discussed previously. The mathematical reason is that the derivative dL/dq has a form

$$\frac{dL}{dq} = -\frac{q(r_+^2 - q^2)^3}{960\pi r_+^2 [r_+^4(r_+^4 + q^4) - q^2\eta(3r_+^4 - 4q^2r_+^2 - q^4)]^2} \left[r_+^8(3r_+^6 + 9r_+^4q^2 + 3r_+^2q^2 + 5q^6) - 2\eta r_+^4(2r_+^8 + 3q^2r_+^6 - 11q^4r_+^4 - 9q^6r_+^2 - 5q^8) - q^4\eta^2(5r_+^6 + 7q^2r_+^4 - 27q^4r_+^2 + 5q^6) \right]. \quad (37)$$

For the smaller η , we can neglect the term contained the parameter η and find the derivative $dL/dq < 0$. For the larger η , the derivative dL/dq is dominated by the second term in the big square-bracket. This leads to

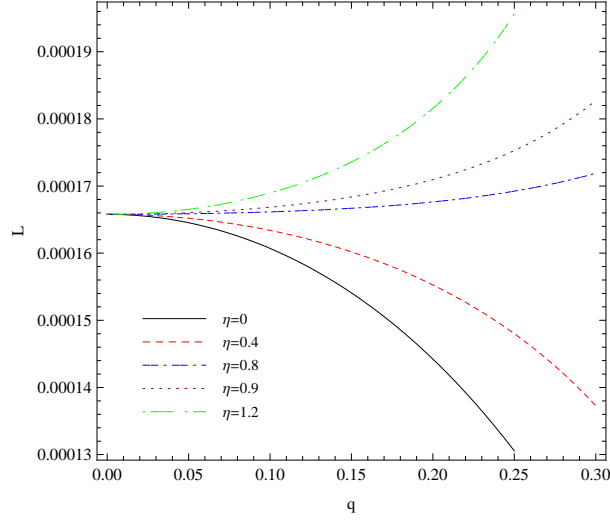


FIG. 5: Variety of the luminosity of Hawking radiation L of scalar particles with the charge q in the Reissner-Nordström black hole for fixed $l = 0$ and different values of η . We set $2M = 1$

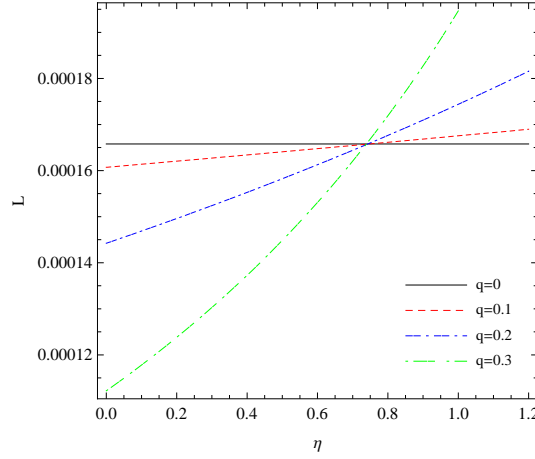


FIG. 6: The dependence of the luminosity of Hawking radiation L of a scalar field on the coupling constant η in the Reissner-Nordström black hole for fixed $l = 0$ and different values of q . We set $2M = 1$.

that $dL/dq > 0$ and we see in Fig.5 that when η is larger the luminosity of Hawking radiation increases with the charge q of the black hole. In terms of Fig.6, we have that the luminosity of Hawking radiation L increases monotonously with the coupling constant η for the all q . Similarly, this effect can be explained by a fact the derivative $dL/d\eta \propto dG/d\eta$ since that the Hawking temperature T_H is independent of the coupling constant η . From Eq.(36), we have

$$\frac{dG}{d\eta} = \frac{4r_+^{10}q^2(r_+^2 - q^2)^2}{[r_+^4(r_+^4 + q^4) - q^2\eta(3r_+^4 - 4q^2r_+^2 - q^4)]^2} > 0. \quad (38)$$

This means that G increases with the increase of η , which leads to the stronger Hawking radiation.

V. SUMMARY

In this Letter, we have studied the greybody factor and Hawking radiation for a scalar field coupling to Einstein's tensor in the background of Reissner-Nordström black hole in the low-energy approximation. We have found that the absorption probability and Hawking radiation depend on the coupling between the scalar field and Einstein's tensor. The presence of the coupling enhances both the absorption probability and Hawking radiation of the scalar field in the black hole spacetime. Moreover, for the weaker coupling, we also find that the absorption probability and Hawking radiation decreases with the charge q of the black hole. It is similar to that of the scalar field without coupling to Einstein's tensor. However, for the stronger coupling, the charge q enhances the absorption probability and Hawking radiation of the black hole, which could provide a way to detect whether there exist a coupling between the scalar field and Einstein's tensor or not. It would be of interest to generalize our study to other black hole spacetimes, such as rotating black holes etc. Work in this direction will be reported in the future.

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